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Stationary State Entanglement of Two Atoms Inside an Optical Cavity under Noise

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Abstract In this paper, the entanglement properties of a system of two atoms inside an optical cavity in a stochastic interaction with field are studied by the Jaynes-Cummings Model. The phase telegraph noise is considered as a noise term and an exact solution to the model is obtained. The solution reveals the resulting decoherence effects of the noise on the entanglement properties of the system. It shows that under the noise the individual atoms do not entangle with the cavity field. However, a strong atom-atom entanglement is observed in a stationary state. It is seen that a relatively strong noise is cooperative in the construction of the steady state atom-atom entanglement.

Keywords Entanglement · Random phase telegraph noise · Concurrence

1 Introduction

Entanglement can display nonlocal correlations between quantum systems that have no classical counterpart. As a physical resource it plays a key role in Quantum Information Processing (QIP) [1–3]. Its preparation is, therefore, a primary goal of this field. So far, static entanglement has been investigated extensively. However, real quantum systems will unavoidably interact with their surrounding environments. The main problem that must be overcome in QIP is decoherence which is an effect that results from the coupling of the system to its surroundings or noise described by the stochastic processes associated with the system.

The influence of noise (jump-type) on the atom-field interactions was first introduced by Burshtein [4–8] in quantum optics. The simplest model of such jump-like processes is the

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two-state random telegraph. Random telegraph models have also been studied extensively by some authors [9, 10]. These jump-like models are very suitable to treat the noise arising from the electromagnetic field fluctuations or from various kinds of collisions or from other external effects.

The interaction of a two-level atom with a single-mode field which makes a single-photon transitions in an ideal cavity is described by the Jaynes-Cummings model (JCM) [11]. The noisy atom-field interactions can be described by generalizing the JCM as studied by some authors [12–15]. In these studies, the incorporation of the noise into the JCM has been treated as the stochastic fluctuations in the atom-field coupling parameter that is assumed to fluctuate in phase or in amplitude. The stochastic fluctuations associated with the coupling parameter in the cavity quantum electrodynamics may presumably arise from several reasons such as due to the source of the single-mode coherent cavity field or due to any variation in the mechanism of the production of Rydberg atom because of the instability in the atomic vapour production. The motion of an ion in a harmonic trap interacting with a standing wave or a traveling wave may introduce another possibility for introducing the JCM with the stochastic fluctuations. Under a particular approximation [16, 17], the equation of the motion for the ion in the trap may reduce to a similar form of the JCM. For this case, the fluctuations in the coupling coefficient can be considered both in the amplitude and the phase of the standing wave. Other possibilities for introducing the stochastic fluctuations in the JCM are the fluctuations of vacuum Rabi frequency and the atom-field coupling coefficient which is known for washing out the trapping states in the micromaser system. Such fluctuations are possible in the case of an electric field generated by rubidium deposits at the cavity coupling holes or the electric field between the adjacent crystal domains in the cavity walls made of niobium.

The employment of the jump-like noise into the ordinary atom-field interaction in the cavity quantum electrodynamics causes some important variations in the dynamics of the interactions. One such important variation is the resulting decoherence effects in the entanglement dynamics of the interaction which will be handled in the present work. Ideally, one hopes that entanglement should be maintained for sufficiently long time for quantum information processing. In this paper, we have shown that it is possible to have an entanglement between two atoms even in the presence of a noise and the noise is found to be cooperative in the construction of the steady state atom-atom entanglement. The noise here is the random-phase telegraph noise which arises from the system itself not from its surrounding environment. This is another type of intrinsic decoherence in the JCM. We studied the interaction of two two-level atoms coupled to a single mode optical cavity under the phase telegraph noise. The entanglement properties between the individual atom and the cavity field as well as between the atoms in the presence of the phase telegraph noise are investigated.

The organization of the paper is as follows; In Sect. 2, the formulation of the problem is presented briefly. The JCM in the presence two atoms with the phase telegraph noise is introduced. The explicit analytical solution of the system of two atoms inside an optical cavity under the phase telegraph noise is obtained. In Sect. 3, the entanglement properties of the system are investigated and the results are discussed. Finally, in Sect. 4, the conclusions are presented.

2 The Formulation of the Problem

2.1 The Model

We have considered the interaction of two two-level atoms described by spin-$1/2$ operators $S_{\pm}^{(1,2)}$, $S_{z}^{(1,2)}$ with a single-mode of quantized radiation field described by the annihilation and the creation operators $a, a^\dagger$, respectively. For the sake of simplicity, it is assumed that the field is in resonance with the atomic transition frequency ω_0. For this case, the Hamiltonian of the system under the rotating wave approximation takes the form of ($\hbar = 1$)

$$H = \omega_0 a^\dagger a + \sum_{i=1,2} \omega_0 S_z^{(i)} + (g^*(t) S_+^{(i)} a + g(t) S_-^{(i)} a^\dagger) \tag{1}$$

where $g(t)$ is the time dependent coupling coefficient between the atom and the field. In order to employ the phase telegraph noise into the problem, $g(t)$ is taken to be completely stochastic, and is defined as

$$g(t) = g_0 e^{i\phi(t)} \tag{2}$$

where g_0 is a positive real constant amplitude and $\phi(t)$ is a stochastic variable fluctuating between different arbitrary phases in a manner of jumps. The jumps are separated by an average time interval called the mean dwell time. $\phi(t)$s in the neighbouring intervals are not correlated. Consequently, the probability of finding ϕ remains the same at any instant of time. Thus, $\phi(t)$ can undergo continuous random change of Markov type which allows us to take the average over the stochastic fluctuations.

2.2 The (Exact) Solution

We consider the system of two atoms which are trapped inside a single mode optical cavity. Initially, the cavity field is set in the vacuum state $|0\rangle$, atom 1 in the excited state $|e\rangle$, and atom 2 in the ground state $|g\rangle$ [18, 19]. In this case, the initial density matrix of the system becomes

$$\rho(0) = |0\rangle\langle 0| \otimes |eg\rangle\langle eg| \tag{3}$$

The exact analytical solution of the system of the time-dependent relevant density matrix in the interaction picture can be get from the method introduced by Joshi [15]

$$\rho(\tau)\exp\left(\frac{\tau}{\tau_0}\right) = \int U(\phi;\tau;0)\rho(0)U^{-1}(\phi;\tau;0)dQ(\phi) + \frac{1}{\tau_0}\int \exp\left(\frac{t}{\tau_0}\right)\int U(\phi;\tau;t)\rho(t)U^{-1}(\phi;\tau;t)dQ(\phi)dt \tag{4}$$

where τ_0 is the mean dwell time and $dQ(\phi) = \frac{d\phi}{2\pi}$. In this integral equation, the statistical average is taken over the stochastic variable $\phi(t)$.

Substituting (3) into (4) with the use of the Laplace Transformation techniques, the time evolution of the density matrix of the system $\rho(t)$ can be obtained as

$$\rho(t) = \rho_{22}(t)|0ge\rangle\langle 0ge| + (\rho_{23}(t)|0ge\rangle\langle 0eg| + h.c.) + \rho_{33}(t)|0eg\rangle\langle 0eg| + \rho_{88}(t)|1gg\rangle\langle 1gg| \tag{5}$$

where

$$\rho_{22}(t) = \frac{1}{8}\{3 + \exp(-t/2T)[(\cosh\mu t/2T + 1/\mu\sinh\mu t/2T) - 4(\cosh\nu t/2T + 1/\nu\sinh\nu t/2T)]\} \tag{6}$$

$$\rho_{23}(t) = \frac{1}{8}\{-1 + \exp(-t/2T)[(\cosh \mu t/2T + 1/\mu \sinh \mu t/2T)]\} \tag{7}$$

$$\rho_{33}(t) = \frac{1}{8}\{3 + \exp(-t/2T)[(\cosh \mu t/2T + 1/\mu \sinh \mu t/2T) + 4(\cosh \nu t/2T + 1/\nu \sinh \nu t/2T)]\} \tag{8}$$

$$\rho_{88}(t) = \frac{1}{4}\{1 - \exp(-t/2T)[(\cosh \mu t/2T + 1/\mu \sinh \mu t/2T)]\} \tag{9}$$

$$\rho_{32}(t) = \rho_{23}(t)^* \tag{10}$$

where $T = \tau_0$, $\mu = \sqrt{1 - 32T^2}$ and $\nu = \sqrt{1 - 8T^2}$.

3 Results and Discussion

We used the (5), the exact solution, to investigate the effects of the phase telegraph noise on the entanglement properties of the system.

By tracing out the degree of freedom of the second atom, one obtains the reduced density matrix $\rho^{af,1}(t)$ of the subsystem for the atom 1 and the field as

$$\rho^{af,1}(t) = \rho_{22}(t)|0g\rangle\langle 0g| + \rho_{33}(t)|0e\rangle\langle 0e| + \rho_{88}(t)|1g\rangle\langle 1g| \tag{11}$$

and repeating it for the first atom to obtain the density matrix for the atom 2 and the field subsystem

$$\rho^{af,2}(t) = \rho_{22}(t)|0e\rangle\langle 0e| + \rho_{33}(t)|0g\rangle\langle 0g| + \rho_{88}(t)|1g\rangle\langle 1g| \tag{12}$$

By tracing out the degree of freedom of the cavity field, one obtains the reduced density matrix $\rho^{aa}(t)$, for the subsystem containing two atoms as

$$\rho^{aa}(t) = \rho_{22}(t)|ge\rangle\langle ge| + (\rho_{23}(t)|ge\rangle\langle eg| + h.c.) + \rho_{33}(t)|eg\rangle\langle eg| + \rho_{88}(t)|gg\rangle\langle gg| \tag{13}$$

The dimensions of the field-atom and the atom-atom subsystems are $2 \otimes 2$. For these systems, the degree of the entanglement can be quantified by the Wootters's concurrence $C(t)$ [20] which is defined as

$$C(t) = \max\{0, \lambda_1(t) - \lambda_2(t) - \lambda_3(t) - \lambda_4(t)\} \tag{14}$$

where $\lambda_1(t) \geq \lambda_2(t) \geq \lambda_3(t) \geq \lambda_4(t)$ are the square roots of the eigenvalues of the matrix $R(t) = \rho(t)(\sigma_y \otimes \sigma_y)\rho(t)^*(\sigma_y \otimes \sigma_y)$. The concurrence varies from 0 for the separable states to 1 for the maximally entangled states.

From the (11) and (12), the degree of the entanglement is calculated and found to zero, $C(t) = 0$. So, there is no entanglement between the field and the individual atoms during the time evolution of the system due to the noise. Thus, we may say that the atoms do not entangle with the cavity field. There may be unnoticeable entanglement between the field and the atoms at the very beginning of the interaction but it disappears in a very short time.

However, the situation is quite different in the atom-atom subsystem entanglement. This can be seen from (13) results. The effects of the random phase telegraph noise on the entanglement properties of the atom-atom subsystem are depicted in Fig. 1.

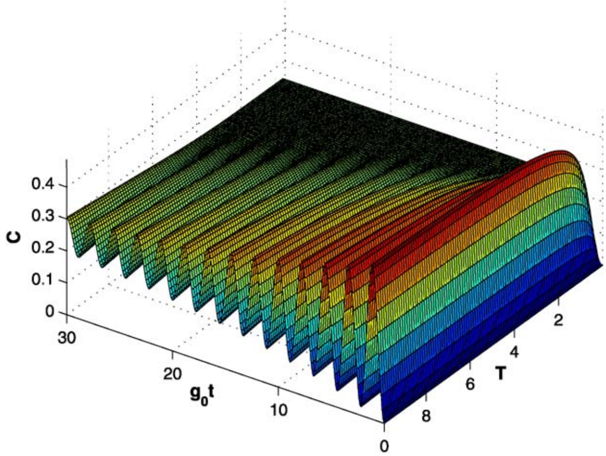

Fig. 1 Concurrence C as the function of the time g_0t and the mean dwell time T

As seen in Fig. 1, the entanglement between the atoms reaches its maximum value in a very short time then decays with oscillations as time passes. On the other hand, the value of mean dwell time T increases, the strength and the lifetime of the entanglement increases. That is, as T increases, the effects of the noise on the interaction weakens. Since the intensity/strength of the noise is determined by the mean dwell time, the decoherence mechanism becomes faster as the mean dwell time is shorter. During the interaction, there is a dephasing mechanism that accounts for the decay in the degree of the entanglement. This mechanism arises from the stochastic phase fluctuations of the atom-field coupling parameter. In the mechanism, the phase fluctuations affect only the dipole or the transverse relaxation mechanism of the system. There is no any type of dissipation in the energy of the system. So, this dephasing mechanism is different from the usual dissipation mechanism which affects both the energy and the coherence of the system. The system is capable of following all these phase changes and can respond them as a decay factor in the entanglement no matter how fast the phase changes are (or no matter how small the mean dwell time is). Therefore, the entanglement dynamics of the atoms is very sensitive to the noise. There is a monotonous relation between the entanglement of the atoms and the mean dwell time of the phase-telegraph noise. But, at the sufficiently small values of the mean dwell time, the decay of the entanglement is stabilized to a finite value and survives in a stationary state. Thus, a sufficiently strong/intense noise (the sufficiently small mean dwell times) will cooperate in the construction of the steady state atom-atom entanglement, not in the destruction of the entanglement. In addition, in the case the value of the mean dwell time goes to infinity $T \to \infty$, the influence of the noise on the atoms weakens and the entanglement dynamics becomes periodic, as expected. See Fig. 2.

It would also be noteworthy to compare this particular dephasing mechanism which causes the revealed properties of the entanglement dynamics with the intrinsic decoherence. The intrinsic decoherence gives rise a destruction of the quantum coherence in the case that the physical properties of the system approach a macroscopic level. In this type of dephasing mechanism, the constants of the motion remain unchanged and hence stationary states remain stationary. Thus, no energy is dissipated from the system as in the present work. Milburn [21] proposed a model for the intrinsic decohorence by modifying Schrödinger evolution. According to this model, the off-diagonal elements of the density matrix of the system are suppressed intrinsically due to the evolution of the system under a stochastic identical unitary transformation giving rise to the quantum decoherence. So, the dephasing

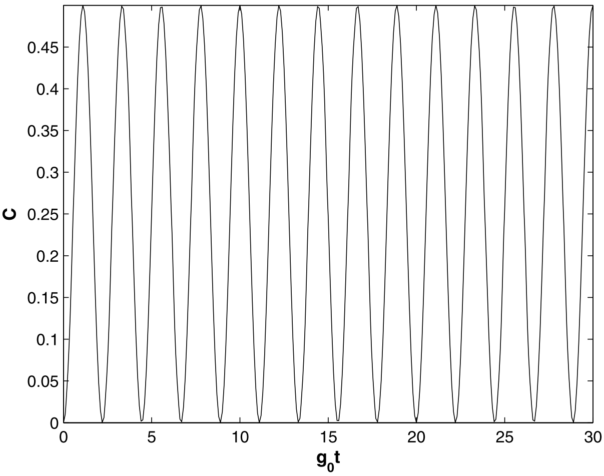

Fig. 2 Concurrence C as the function of the time $g_0 t$. $T \to \infty$

mechanisms in both cases display similar physical features (no energy dissipation and the decay of quantum coherences).

Finally, it should be pointed out that the noise we considered in this study is not controllable. It is completely due to the stochastic behavior of the system itself, not due to an environment effect. This is another type of intrinsic decoherence in the JCM. There are some works devoted to the environmental noise. For example, one is about preventing or minimizing the influence of environmental noise in quantum information processing [22–24]. But, instead of attempting to shield the system from the environmental noise, Plenio and Huelge used a white noise to generate a controllable entanglement by incoherent sources [25]. The entanglement dynamics in their work displays a similar character with that of ours. The noise plays a constructive role in quantum information processing but the entanglement arises from the controllable situation. Similar aspects have also been considered elsewhere [26, 27]. Hence, the revealed properties of the entanglement of the system under the random phase telegraph noise are uncontrollable (intrinsic) and unaffected by the surrounding environment. Since the fluctuations in the system are quite random, the entanglement equivalently the information in the system fluctuates randomly.

4 Conclusion

The system of two two-level atoms coupled to a single mode optical cavity under the phase telegraph noise was investigated analytically. The effects of the phase telegraph noise on the entanglement dynamics of the system were explored. We have shown that there is no entanglement between the individual atoms and the field in time due to the noise. The phase telegraph noise does not permit the atoms to be entangled with the cavity field. However, the atom-atom entanglement is obtainable and strong under the phase telegraph noise compared to the field-atom entanglement. We have seen that the entanglement of the atom-atom subsystem is very sensitive to the noise. The system can almost follow all phase changes of the noise and can respond them as a decay in the degree of the entanglement, no matter how fast the phase changes are, or no matter how small the mean dwell time is. There is a monotonous relationship between the entanglement of the atoms and the mean dwell time of the phase-telegraph noise. At the sufficiently small values of the mean dwell time, the decay of the entanglement stabilizes to a finite value and survives in a stationary state. The sufficiently strong/intense noise, for the sufficiently small mean dwell times, is cooperative in the construction of the steady state atom-atom entanglement instead of the destruction of the entanglement in a uncontrollable manner.

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